TUTORIAL II (LINEAR ALGEBRA)

MS 103: Mathematics II

Course Instructor: Parama Dutta

---

1. Prove that the followings are linear transformations

   (a) $T : \mathbb{R}^3 \mapsto \mathbb{R}^2$ defined by $T(a_1, a_2, a_3) = (a_1, -a_2, 2a_3)$.
   (b) $T : \mathbb{R}^2 \mapsto \mathbb{R}^3$ defined by $T(a_1, a_2) = (a_1 + a_2, 0, 2a_1 - a_2)$.
   (c) $T : M_{2\times3}(\mathbb{F}) \mapsto M_{2\times2}(\mathbb{F})$ defined by

$$T\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \mapsto \begin{bmatrix} 2a_{11} - a_{12} & a_{13} + 2a_{12} \\ 0 & 0 \end{bmatrix}.$$

   (d) $T : P_2(\mathbb{R}) \mapsto P_3(\mathbb{R})$ defined by $T(f(x)) = xf(x) + f'(x)$.
   (e) $T : M_{n\times n}(\mathbb{F}) \mapsto \mathbb{F}$ defined by $T(A) = trace(A)$.

2. Let $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ is a function. For each of the following parts, state why T is not linear transformation.

   (a) $T(a_1, a_2) = (1, a_2)$.
   (b) $T(a_1, a_2) = (a_1, a_2^2)$.
   (c) $T(a_1, a_2) = (sin a_1, 0)$.
   (d) $T(a_1, a_2) = (|a_1|, a_2)$.
   (e) $T(a_1, a_2) = (a_1 + 1, a_2)$.

3. Suppose that $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ is linear. If $T(1, 0) = (1, 4)$ and $T(1, 1) = (2, 5)$ then what is $T(2, 3)$?

4. Let $V = C(\mathbb{R})$, the vector space of continuous real-valued functions on $\mathbb{R}$. Let $a, b \in \mathbb{R}$, $a \leq b$. Define $T : V \mapsto R$ by $T(f) = \int_a^b f(t)dt$ for all $f \in V$. Then prove that T is a linear transformation.

5. Find the row reduced echelon form of the following matrices and hence find the rank of the matrices.

   (a) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$

   (d) $\begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 2 & -3 & 4 & 9 \\ 1 & 0 & -1 & 1 & 1 \\ 3 & -1 & 1 & 0 & -1 \\ -1 & 1 & 0 & 2 & 9 \\ 3 & 1 & 0 & 3 & 9 \end{bmatrix}$

6. Reduce the following matrices in its normal form and hence find the rank of the matrices.

   (a) $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \\ 3 & 1 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{bmatrix}$

   (d) $\begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$

8. Find inverse of the following matrices (if exists) by Gauss Jordan method.

(a) $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 4 & 3 & 2 \\ 3 & 6 & 5 & 2 \\ 2 & 5 & 2 & -3 \\ 4 & 5 & 14 & 14 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 6 \end{bmatrix}$

****

SOLUTIONS

5. (a) 3 (b) 2 (c) 2 (d) 3 (e) 4

6. (a) 2 (b) 3 (c) 2 (d) 2 (e) 3

7. (a) $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -23 & 29 & -\frac{64}{5} & -\frac{18}{5} \\ 10 & -12 & \frac{26}{5} & \frac{7}{5} \\ 1 & -2 & \frac{6}{5} & \frac{2}{5} \\ 2 & -2 & \frac{3}{5} & \frac{1}{5} \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 2 & 1 & 3 \\ 1 & 1 & -1 & -2 \\ 1 & 2 & 0 & 1 \\ -1 & 1 & 2 & 6 \end{bmatrix}$

(d) not exist (e) not exist